

Higgs Boson in RG running Inflationary Cosmology

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An intriguing hypothesis is that gravity may be non-perturbatively renormalizable via the notion of asymptotic safety. We show that the Higgs sector of the SM minimally coupled to asymptotically safe gravity can generate the observed near scale-invariant spectrum of the Cosmic Microwave Background through the curvaton mechanism. The resulting primordial power spectrum places an upper bound on the Higgs mass, which for canonical values of the curvaton parameters, is compatible with the recently released Large Hadron Collider data.

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Weinberg has suggested that the effective description of a quantum gravitational theory may be nonperturbatively renormalizable through the notion of asymptotic safety (AS) [1]. In such a scenario the renormalization group (RG) flows approach a fixed point in the ultraviolet (UV) limit, and a finite dimensional critical surface of trajectories evolves to this point at short distance scales [2]. Such a fixed point was found in the Einstein-Hilbert truncation [3], and the scenario was studied extensively in the literature [4] (for recent reviews see [5]).

Inflationary cosmology is the most promising candidate theory for describing the early universe [6]. The paradigm solves the homogeneity, flatness, horizon and unwanted relic problems. It also predicts a nearly scale-invariant primordial power spectrum, in agreement with the data from modern cosmological experiments [7]. However, the model generally requires an as of yet unobserved scalar field. The Higgs boson is a scalar field predicted by the SM of particle physics and exciting evidence for its existence has recently been released from the Large Hadron Collider (LHC) experiment [8–10]. In the past, it was hoped that the Higgs field might play a dual role as the inflaton, but the corresponding energy scale is much lower than what inflationary cosmology typically requires. Consequently, a model of Higgs inflation was proposed in which the Higgs field is non-minimally coupled to Einstein gravity [11] (see earlier attempt in [12]). It was soon realized that such non-minimal coupling leads to an energy scale for unitarity violation which is expected to be larger than the inflationary scale; otherwise, the effective field description would fail (for example, see [13] for detailed discussions). Therefore, difficult challenges exist for models which attempt to use the Higgs field to drive inflation.

In this paper, we propose that the Higgs boson may play an important role in the early inflationary universe if the gravitational theory is asymptotically safe. In the frame of AS gravity, the gravitational constant G and cosmological constant Λ are running along with the energy scale, and thus vary throughout the cosmological evolution. It has been argued that if there are no intermediate energy scales between the SM and AS scales, the mass of

the Higgs boson is predicted to be $m_H = 126$ GeV with only several GeV uncertainty [14]. We find a suitable inflationary solution can be obtained in a cosmological system which contains a Higgs boson and AS gravity, along the lines of [15]. In this model, there are effectively two scalar degrees of freedom, one being the adiabatic mode and the other being an iso-curvature mode. We find the corresponding perturbation theory leads to both the primordial power spectrum for the curvature perturbation and the entropy perturbation. When the cutoff scale runs lower than a critical value, inflation abruptly ends and the Higgs field can give rise to a reheating phase. During this phase, the fluctuations seeded by the Higgs field can be converted into the curvature perturbation through the curvaton mechanism [16, 17]. We derive a relation between the spectral index of the primordial power spectrum and the Higgs mass. We confront this relation with the latest cosmological observations and collider experiment data, and find they are consistent under a group of canonical values of curvaton parameters.¹

Consider the SM of particle physics minimally coupled to gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}^{SM} \right]. \quad (1)$$

In this AS gravity frame, the gravitational constant G and the cosmological constant Λ vary along the cutoff scale p . The running behaviors are approximately described by

$$G(p)^{-1} \simeq G_N^{-1} + \xi_G p^2, \quad \Lambda(p) \simeq \Lambda_{IR} + \xi_\Lambda p^2, \quad (2)$$

where G_N and Λ_{IR} are the values of gravitational constant and cosmological constant in the IR limit. The coefficients ξ_G and ξ_Λ are determined by the physics near the UV fixed point of RG flows in AS gravity.

The scalar sector of the SM contains the Higgs boson. We use the unitary gauge for the Higgs boson $H = \frac{h}{\sqrt{2}}$

¹ In this Letter, we will work with the reduced Planck mass, $M_p = 1/\sqrt{8\pi G_N}$, where G_N is the gravitational constant in the IR limit, and adopt the mostly-plus metric sign convention $(-, +, +, +)$.

and neglect all gauge interactions for the time being. In this case, the Lagrangian of the Higgs field is given by,

$$\mathcal{L}^{SM} \supseteq -\frac{1}{2}\partial_\mu h \partial^\mu h - V(h), \quad (3)$$

where $V(h)$ is the potential of the Higgs field, which is typically in form of $\frac{\lambda}{4}(h^2 - v^2)^2$.

Varying the Lagrangian with respect to the metric, one derives the generalized Einstein equation,

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G(T_{\mu\nu}^{SM} + T_{\mu\nu}^{AS}). \quad (4)$$

Here the RG running of G can effectively contribute to the stress energy tensor through $T_{\mu\nu}^{AS} = (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square)(8\pi G)^{-1}$, where we have introduced the covariant derivative ∇_μ and the operator $\square \equiv -g^{\mu\nu} \nabla_\mu \nabla_\nu$. The Higgs field h obeys the Klein-Gordon equation. Additionally, the running of cutoff scale is controlled by the Bianchi identity, which requires, $(R - 2\Lambda)\frac{\nabla_\mu G}{G} + 2\nabla_\mu \Lambda = 0$. Consequently, the dynamics of this cosmological system are completely determined.

We now turn our attention to early universe inflationary solutions. This system is most easily studied by making a conformal transformation,

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = \frac{G_N}{G}, \quad (5)$$

where Ω^2 is the conformal factor. We also introduce a new scalar field ϕ , defined by

$$\phi \equiv -\frac{\sqrt{6}M_p}{2} \ln \frac{G_N}{G}, \quad (6)$$

where $M_p = \frac{1}{\sqrt{8\pi G_N}}$.

The original system, therefore, is equivalently described in terms of two scalar fields minimally coupled to Einstein gravity without RG running,

$$\mathcal{L} = \frac{\tilde{R}}{16\pi G_N} - \frac{1}{2}(\tilde{\nabla}\phi)^2 - \frac{e^{2b(\phi)}}{2}(\tilde{\nabla}h)^2 - \tilde{V}(\phi, h), \quad (7)$$

where the factor $b(\phi) \equiv \frac{\phi}{\sqrt{6}M_p}$ and $\tilde{V}(\phi, h) = U(\phi) + e^{4b}V(h)$, with $U(\phi) \simeq 8\pi M_p^4 \left[\frac{\xi\Lambda}{\xi_G} \left(1 - e^{2b(\phi)} \right) + G_N \Lambda_{IR} e^{2b(\phi)} \right]$. Note that, the form of ϕ 's potential is derived from the RG running of the AS gravity [15]. The last term of $U(\phi)$ is proportional to $G_N \Lambda_{IR}$. Substituting the observed values of G_N and Λ_{IR} , we find this term is insignificant throughout the past cosmological evolution. Hence, we make the approximation

$$U(\phi) \simeq 8\pi M_p^4 \frac{\xi\Lambda}{\xi_G} \left[1 - e^{2b(\phi)} \right], \quad (8)$$

which is a sufficiently flat inflationary potential in the regime where $b(\phi) \ll -1$.

We denote the frame proceeding the conformal transformation as the Einstein frame (despite the non-canonical form of the h kinetic term). Substitution of the flat Friedmann-Robertson-Walker (FRW) metric, $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$, leads to the Friedmann equations: $H^2 = \frac{1}{3M_p^2}(\frac{\dot{\phi}^2}{2} + \frac{e^{2b}\dot{h}^2}{2} + \tilde{V})$ and $\dot{H} = -\frac{1}{2M_p^2}(\dot{\phi}^2 + e^{2b}\dot{h}^2)$, where we have defined the Hubble parameter $H \equiv \frac{\dot{a}}{a}$ and the dot denotes the time derivative in the Einstein frame. The coupled Klein-Gordon equations for the two scalars are: $\ddot{\phi} + 3H\dot{\phi} + \tilde{V}_{,\phi} = b_{,\phi}e^{2b}\dot{h}^2$, $\ddot{h} + (3H + 2b_{,\phi}\dot{\phi})\dot{h} + e^{-2b}\tilde{V}_{,h} = 0$.

To search for a successful inflationary solution, we introduce a series of slow roll parameters,

$$\epsilon_\phi = \frac{\dot{\phi}^2}{2M_p^2 H^2}, \quad \epsilon_h = \frac{e^{2b}\dot{h}^2}{2M_p^2 H^2}, \quad \eta_{IJ} = \frac{\tilde{V}_{,IJ}}{3H^2}. \quad (9)$$

The subscript “ $_I$ ” denotes the derivative with respect to the I th-field (with I being ϕ or h). During inflation, these parameters are required to be less than unity. However, the key parameters need to yield a successful inflationary background are associated with the scalar ϕ , i.e., ϵ_ϕ and $\eta_{\phi\phi}$. This is because the potential for ϕ is flat in the regime $\phi \ll -M_p$ and correspondingly the parameters related to the Higgs boson h are suppressed by the small-valued factor e^{2b} .

The background dynamics is determined by the following solutions (under slow roll approximation),

$$\dot{\phi} \simeq -\frac{U_{,\phi}}{3H}, \quad \dot{h} \simeq -\frac{e^{2b}V_{,h}}{3H}, \quad H^2 \simeq \frac{U}{3M_p^2}. \quad (10)$$

Inflation ends when $\epsilon_\phi = 1$. Combining this condition with the background solution for $\dot{\phi}$ in (10), we find the value of ϕ at the end of inflation: $\phi_f \simeq -0.56M_p$. The number of e-folding of inflation is given by $\mathcal{N} = -\int_i^f \frac{U d\phi}{M_p^2 U_{,\phi}}$, so that

$$\mathcal{N}(\phi) \simeq \frac{3}{2}e^{-2b(\phi)} + 3b(\phi) - 1.68. \quad (11)$$

To obtain $\mathcal{N} = 60$, we require the initial inflaton value to be, $\phi_i \simeq -4.7M_p$, which lies in the regime where RG flows of AS gravity have approached the UV fixed point.

Eventually, the slow roll conditions are violated, when ϕ reaches ϕ_f . Consequently, ϕ enters a period of fast roll, and finally approaches $\phi = 0$ at which point the AS gravity reduces to traditional Einstein gravity. We suggest two possible reheating mechanism. One is that the inflaton ϕ decays to radiation, driving the universe to a phase of thermal expansion directly; the other possibility is that the universe reheats only after the energy scale drops sufficiently so that the SM Higgs boson is responsible for the reheating process. Based on the assumption of instant reheating, the reheating temperature is approximately given by $T_{re} \simeq (\frac{2\lambda}{\pi^2 g_d})^{\frac{1}{4}} h_{re}$ where h_{re} is the value of h at the reheating surface, and $g_d \simeq 106.75$ is the number of degrees of freedom of the SM. After

inflation but before reheating, h oscillates along the potential $V \sim \lambda h^4$. Thus the value of h can be restricted to $v < |h_{re}| < h_*$ with h_* being the value of the Higgs boson at the moment of Hubble crossing which we discuss below.

During inflation, the background dynamics are not affected by the Higgs field, however its quantum fluctuations are able to source a nearly scale-invariant entropy perturbation. In a two field inflationary model, we decompose the field variables into a background part and fluctuations: $\phi \rightarrow \phi + \delta\phi$ and $h \rightarrow h + \delta h$. The field fluctuations combine to give adiabatic and iso-curvature modes as: $\delta\sigma = \cos\theta\delta\phi + \sin\theta e^b\delta h$, $\delta s = -\sin\theta\delta\phi + \cos\theta e^b\delta h$ with the trajectory angle being defined by $\cos\theta = \frac{\dot{\phi}}{\dot{\sigma}}$ and $\sin\theta = \frac{e^b\dot{h}}{\dot{\sigma}}$ [18]. To take into account the metric fluctuation (the gravitational potential Φ) we introduce the canonical perturbation variables, $v_\sigma = a(\delta\sigma + \frac{\dot{\sigma}}{H}\Phi)$ and $v_s = a\delta s$, which characterize gauge-invariant adiabatic and iso-curvature perturbations. Up to leading order in the slow roll approximations, these two variables obey the perturbation equations [19]: $v''_{\sigma(s)} + (k^2 - \frac{a''}{a})v_{\sigma(s)} \simeq 0$, where the prime denotes differentiation with respect to conformal time, $\tau \equiv \int \frac{dt}{a}$. Solving this equation in the inflationary phase, we can obtain nearly scale-invariant primordial power spectra for adiabatic and iso-curvature perturbations, and their corresponding amplitudes are $|\delta\sigma| \simeq |\delta s| \simeq \frac{H_*}{2\pi}$ at the Hubble crossing moment t_* . Hence, the amplitudes of the field fluctuations are,

$$|\delta\phi_*| \simeq \frac{H_*}{2\pi}, \quad |\delta h_*| \simeq \frac{H_*}{2\pi e^{b_*}}, \quad (12)$$

at the moment of Hubble crossing.

When inflation ends, ϕ rapidly approaches the IR limit of the AS gravity. Because ϕ generally couples to other matter fields through the conformal factor Ω^2 , we expect radiation to be produced following the inflationary phase. Recall that, the Higgs boson can survive during inflation due to the slow roll conditions and then starts to oscillate along the λh^4 potential. Consequently, the universe is dominated by both the radiation and the Higgs boson after inflation. This process is analogous to the familiar curvaton scenario. Instead of a matter-like curvaton oscillation with $w = 0$ as studied in [16, 20], a λh^4 potential yields an effective equation of state for the Higgs boson, which is the same as radiation with $w_h = \frac{1}{3}$ [21]. Thus, this generalized curvaton mechanism [22] may be used to generate the primordial curvature perturbation in agreement with the current CMB measurements [7].

We begin by writing down the relation between the curvaton fluctuation δh and its curvature perturbation ζ_h . Choosing the spatially flat slice for the Higgs curvaton, one finds $\rho_h = \bar{\rho}_h e^{3(1+w_h)\zeta_h}$ in the neighborhood of the curvaton reheating hypersurface. Consider the curvaton perturbation generated from vacuum fluctuations inside the Hubble radius. These fluctuations satisfy a Gaussian

distribution at the Hubble exit. In general, the Hubble-crossing value of the Higgs boson h_* can be related to the initial amplitude of curvaton oscillation h_o through a model-dependent function $h_o = g(h_*)$. For example, in the present model, if the curvaton starts to oscillate immediately after inflation, $h_o \simeq h_*$; however, if there is a short slow rolling behavior for h following inflation, $h_o \simeq \frac{11}{12}h_*$. In this case, the curvature perturbation of the Higgs field in the oscillating phase is given by,

$$\zeta_h = \frac{\delta\rho_h}{3(1+w_h)\rho_h} \simeq q_h \frac{\delta h_*}{h_*}, \quad (13)$$

with $q_h \equiv \frac{h_o h_*}{h_o^2 - v^2}$. The coefficient q_h can be further simplified as $q_h \simeq \frac{h_*}{h_o}$ when $|h_o| \gg v$ if curvaton reheating occurs at energy scales higher than the energy scale of the SM.

We now need to relate ζ_h to ζ . In the sudden decay approximation, the relation can be computed analytically. Consider the case that the Higgs boson decays on a uniform total density hypersurface. On this slice we have $\rho_h + \rho_r = \rho_T$ where ρ_r and ρ_T denote the energy density of radiation and that of the total system, respectively. Making use of the expression for the curvature perturbation on a uniform density slice, we find $\rho_r = \bar{\rho}_r e^{4(\zeta_r - \zeta)}$ and $\rho_h = \bar{\rho}_h e^{3(1+w_h)(\zeta_h - \zeta)}$ during curvaton oscillation. As a consequence, ζ and ζ_h are related on the reheating hypersurface as follows,

$$(1 - \Omega_h)e^{4(\zeta_r - \zeta)} + \Omega_h e^{3(1+w_h)(\zeta_h - \zeta)} = 1, \quad (14)$$

where $\Omega_h = \rho_h/\rho_T$ is the dimensionless density parameter for the curvaton. For the curvaton mechanics to succeed, we must assume that the fluctuation ζ_r seeded by the inflaton field is negligible. We will address this concern below and now turn our attention to the Higgs curvaton. Therefore, we have

$$\zeta = q_T \zeta_h, \quad q_T = \frac{3(1+w_h)\Omega_h}{4 - (1-3w_h)\Omega_h}, \quad (15)$$

and in our explicit case, $q_T = \Omega_h$ at the curvaton decay surface.

Combining Eqs. (13) and (15) and the field fluctuation (12), we obtain the primordial power spectrum of curvature perturbation seeded by the Higgs boson,

$$P_\zeta = \frac{q_h^2 q_T^2}{4\pi^2 e^{2b_*}} \frac{H_*^2}{h_*^2}. \quad (16)$$

We see from (16), that the final curvature perturbation depends on the five parameters: q_h , q_T , H_* , h_* , and e^{b_*} . Compared with the usual curvaton mechanism, our model contains a new parameter e^{b_*} due to the conformal transformation made in Eq. (5). However, since the background dynamics of inflation are driven by the RG running of AS gravity, we find $e^{b_*} \simeq 0.15$ for observable perturbation modes at Hubble exit. Moreover, we have specified the curvaton to be the Higgs boson and hence,

the potential is of an explicit form, and thus $q_h \sim 1$. Since the latest CMB data reveals $P_\zeta \simeq 2.4 \times 10^{-9}$, we can deduce the useful relation $H_* \simeq 4.5 \times 10^{-5} \frac{h_*}{q_T}$. Subsequently, we are left to constrain q_T , H_* and h_* by various theoretical and observational requirements. In the following, we calculate the tensor-to-scalar ratio, the spectral index, and the reheating temperature, respectively, and then constrain the remaining parameters.

The calculation of primordial tensor perturbations is identical to that of ordinary inflationary models, and thus the tensor power spectrum is nearly scale-invariant, given by $P_t = \frac{2H_*^2}{\pi^2 M_p^2}$. The tensor-to-scalar ratio: $r \equiv \frac{P_t}{P_\zeta} = \frac{8e^{2b_*} h_*^2}{q_h^2 q_T^2 M_p^2}$. Since $e^{b_*} \simeq 0.15$ and $q_h \sim 1$, we obtain $r \simeq \frac{0.2h_*^2}{q_T^2 M_p^2}$. According to the latest CMB data, r is required to be less than 0.36 [7], so that $h_* < 2q_T M_p$. A further constraint comes from the curvaton condition that the contribution of inflaton fluctuation to curvature perturbation should be negligible. This condition requires $q_T \zeta_h \gg (1 - q_T) \zeta_r$. Since ζ_r corresponds to the radiation perturbation, inherited from inflaton fluctuation, $\zeta_r \simeq \frac{H_*}{2\pi\sqrt{2\epsilon_*} M_p}$. Choosing a group of canonical parameter values, this condition requires $h_* < q_T M_p$ which is close to the observational bound provided by the tensor-to-scalar ratio. Combining this inequality and the expression of H_* derived from P_ζ , one obtains a constraint on the inflationary Hubble parameter $H_* < 4.5 \times 10^{-5} M_p$.

During inflation the Hubble parameter is approximately constant and the field fluctuations are approximately conserved after Hubble exit. The spectral tilt $n_h \equiv 1 + \frac{d \ln P_h}{d \ln k}$ of the primordial perturbations at the moment of horizon crossing is:

$$n_h \simeq 1 - \frac{2\epsilon_*^{\frac{1}{2}}}{\sqrt{3}} - 2\epsilon_* + \frac{q_h^2 q_T^2 m_H^2}{4\pi^2 v^2 P_\zeta}, \quad (17)$$

where in the r.h.s. we have used the expression for the primordial power spectrum. The Higgs mass is determined by $m_H = \sqrt{2}\lambda v$. In this model the inflaton potential is explicitly defined, and yields $\epsilon \simeq 1.6 \times 10^{-4}$ at the beginning of inflation. Thus, the spectral index can be simplified as $n_h \simeq 0.985 + \frac{q_h^2 q_T^2 m_H^2}{4\pi^2 v^2 P_\zeta}$ for perturbation modes which exit the Hubble radius during the first several efolds. As a consequence, the mass of Higgs boson acquires an upper bound from the CMB measurement:

$$m_H < 3 \times 10^{-5} \frac{v}{q_T}. \quad (18)$$

For the SM Higgs, $v \simeq 246 \text{ GeV}$, and if q_T is smaller than 5.7×10^{-5} , we conclude that the Higgs mass has to be less than 129 GeV.

In this Letter, we proposed a new inflation model in the context of an asymptotically safe gravitational cosmology with renormalization group running gravitational constant. The theory is minimally coupled to the Standard Model sector. As observed in [14], if there are no

intermediate energy scales between the SM and the AS scales, the mass of the Higgs boson is determined by a fixed point and is approximately 126 GeV. This prediction is coincident with the recent results announced from the ATLAS and CMS experiments, indicating that the Higgs mass is in the range 116 – 131 GeV (ATLAS[9]) or 115 – 127 GeV (CMS[10]), with other masses excluded at the 95% confidence level. We have demonstrated that a sufficiently long inflationary solution can be obtained in this scenario due to the RG running of gravitational and cosmological constants, where the Higgs boson plays the role of a curvaton which is responsible for generating the primordial curvature perturbation. From the analysis of linear perturbations, we find this model favors low energy scale inflation. Furthermore, if the occupation of curvaton density at the reheating surface is fixed, then observational constraints on the spectral index imply an upper bound on the Higgs mass. After a fine tuning of the curvaton parameters, the model is consistent with recent LHC data. A complete data fitting of our model will appear in the forthcoming work [23].

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